LESSON 0-1 Plan for Problem Solving

Objective

- Use the four-step problem-solving plan.

NewVocabulary

four-step problem-solving plan
defining a variable

Common Core State Standards

Mathematical Practices

1 Make sense of problems and persevere in solving them.

Using the **four-step problem-solving plan** can help you solve any word problem.

KeyConcept Four-Step Problem Solving Plan

Step 1 **Understand** the Problem.

Step 2 **Plan** the Solution.

Step 3 **Solve** the Problem.

Step 4 **Check** the Solution.

Each step of the plan is important.

Step 1 Understand the Problem

To solve a verbal problem, first read the problem carefully and explore what the problem is about.

- Identify what information is given.
- Identify what you need to find.

Step 2 Plan the Solution

One strategy you can use is to write an equation. Choose a variable to represent one of the unspecified numbers in the problem. This is called **defining a variable**. Then use the variable to write expressions for the other unspecified numbers in the problem.

Step 3 Solve the Problem

Use the strategy you chose in Step 2 to solve the problem.

Step 4 Check the Solution

Check your answer in the context of the original problem.

- Does your answer make sense?
- Does it fit the information in the problem?

Example 1 Use the Four-Step Plan

FLOORS Ling's hallway is 10 feet long and 4 feet wide. He paid $200 to tile his hallway floor. How much did Ling pay per square foot for the tile?

Understand We are given the measurements of the hallway and the total cost of the tile. We are asked to find the cost of each square foot of tile.

Plan Write an equation. Let f represent the cost of each square foot of tile. The area of the hallway is 10×4 or 40 ft^2.

40	times	the cost per square foot	equals	200.
40	$\cdot$	f	$=$	200

Solve $40 \cdot f = 200$. Find f mentally by asking, "What number times 40 is 200?"

$f = 5$

The tile cost $5 per square foot.

Check If the tile costs $5 per square foot, then 40 square feet of tile costs $5 \cdot 40$ or $200. The answer makes sense.

When an exact value is needed, you can use estimation to check your answer.

Example 2 Use the Four-Step Plan

TRAVEL Emily's family drove 254.6 miles. Their car used 19 gallons of gasoline. Describe the car's gas mileage.

Understand We are given the total miles driven and how much gasoline was used. We are asked to find the gas mileage of the car.

Plan Write an equation. Let G represent the car's gas mileage.

gas mileage = number of miles ÷ number of gallons used

$G = 254.6 \div 19$

Solve $G = 254.6 \div 19$

$= 13.4$ mi/gal

The car's gas mileage is 13.4 miles per gallon.

Check Use estimation to check your solution.

260 mi ÷ 20 gal = 13 mi/gal

Since the solution 13.4 is close to the estimate, the answer is reasonable.

LESSON 0-2 Real Numbers

Objective

- Classify and use real numbers.

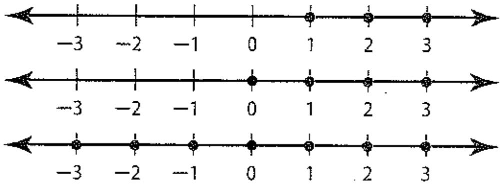
NewVocabulary

positive number
negative number
natural number
whole number
integer
rational number
square root
principal square root
perfect square
irrational number
real number
graph
coordinate

A number line can be used to show the sets of natural numbers, whole numbers, integers, and rational numbers. Values greater than 0, or **positive numbers**, are listed to the right of 0, and values less than 0, or **negative numbers**, are listed to the left of 0.

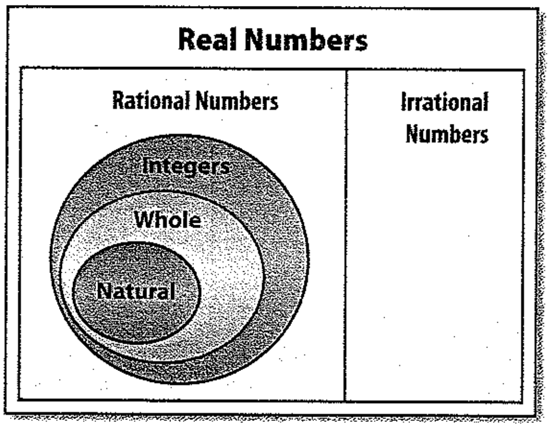
natural numbers: 1, 2, 3, ...
whole numbers: 0, 1, 2, 3, ...
integers: ..., −3, −2, −1, 0, 1, 2, 3, ...

rational numbers: numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$



A **square root** is one of two equal factors of a number. For example, one square root of 64, written as $\sqrt{64}$, is 8 since $8 \cdot 8$ or 8^2 is 64. The nonnegative square root of a number is the **principal square root.** Another square root of 64 is −8 since $(-8) \cdot (-8)$ or $(-8)^2$ is also 64. A number like 64, with a square root that is a rational number, is called a **perfect square.** The square roots of a perfect square are rational numbers.

A number such as $\sqrt{3}$ is the square root of a number that is not a perfect square. It cannot be expressed as a terminating or repeating decimal; $\sqrt{3} \approx 1.73205....$ Numbers that cannot be expressed as terminating or repeating decimals, or in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called **irrational numbers.** Irrational numbers and rational numbers together form the set of **real numbers.**



Example 1 Classify Real Numbers

Name the set or sets of numbers to which each real number belongs.

a. $\frac{5}{22}$

Because 5 and 22 are integers and $5 \div 22 = 0.2272727...$ or $0.2\overline{27}$, which is a repeating decimal, this number is a rational number.

b. $\sqrt{81}$

Because $\sqrt{81} = 9$, this number is a natural number, a whole number, an integer, and a rational number.

c. $\sqrt{56}$

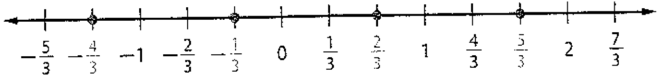
Because $\sqrt{56} = 7.48331477...$, which is not a repeating or terminating decimal, this number is irrational.

To **graph** a set of numbers means to draw, or plot, the points named by those numbers on a number line. The number that corresponds to a point on a number line is called the **coordinate** of that point. The rational numbers and the irrational numbers complete the number line.

Example 2 Graph and Order Real Numbers

Graph each set of numbers on a number line. Then order the numbers from least to greatest.

a. $\left\{\frac{5}{3}, -\frac{4}{3}, \frac{2}{3}, -\frac{1}{3}\right\}$

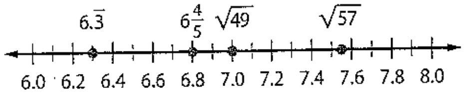


From least to greatest, the order is $-\frac{4}{3}, -\frac{1}{2}, \frac{2}{3}$, and $\frac{5}{3}$.

b. $\left\{6\frac{4}{5}, \sqrt{49}, 6.\overline{3}, \sqrt{57}\right\}$

Express each number as a decimal. Then order the decimals.

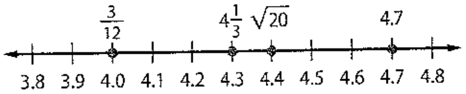
$6\frac{4}{5} = 6.8 \qquad \sqrt{49} = 7 \qquad 6.\overline{3} = 6.33333333\ldots \qquad \sqrt{57} = 7.5468344\ldots$



From least to greatest, the order is $6.\overline{3}$, $6\frac{4}{5}$, $\sqrt{49}$, and $\sqrt{57}$.

c. $\left\{\sqrt{20}, 4.7, \frac{12}{3}, 4\frac{1}{3}\right\}$

$\sqrt{20} = 4.47213595\ldots \qquad 4.7 = 4.7 \qquad \frac{12}{3} = 4.0 \qquad 4\frac{1}{3} = 4.33333333\ldots$



From least to greatest, the order is $\frac{12}{3}$, $4\frac{1}{3}$, $\sqrt{20}$, and 4.7.

Any repeating decimal can be written as a fraction.

Example 3 Write Repeating Decimals as Fractions

Write $0.\overline{7}$ as a fraction in simplest form.

Step 1

$N = 0.777\ldots$ Let N represent the repeating decimal.

$10N = 10(0.777\ldots)$ Since only one digit repeats, multiply each side by 10.

$10N = 7.777\ldots$ Simplify.

Step 2 Subtract N from $10N$ to eliminate the part of the number that repeats.

$$\begin{array}{r} 10N = 7.777\ldots \\ \underline{-(N = 0.777\ldots)} \end{array}$$

$9N = 7$ Subtract.

$\frac{9N}{9} = \frac{7}{9}$ Divide each side by 9.

$N = \frac{7}{9}$ Simplify.

StudyTip

Perfect Squares Keep a list of perfect squares in your notebook. Refer to it when you need to simplify a square root.

Perfect squares can be used to simplify square roots of rational numbers.

KeyConcept Perfect Square

Words Rational numbers with square roots that are rational numbers.

Examples 25 is a perfect square since $\sqrt{25} = 5$.

144 is a perfect square since $\sqrt{144} = 12$.

Example 4 Simplify Roots

Simplify each square root.

a. $\sqrt{\frac{4}{121}}$

$$\sqrt{\frac{4}{121}} = \sqrt{\left(\frac{2}{11}\right)^2} \qquad 2^2 = 4 \text{ and } 11^2 = 121$$

$$= \frac{2}{11} \qquad \text{Simplify.}$$

b. $-\sqrt{\frac{49}{256}}$

$$-\sqrt{\frac{49}{256}} = -\sqrt{\left(\frac{7}{16}\right)^2} \qquad 7^2 = 49 \text{ and } 16^2 = 256$$

$$= -\frac{7}{16}$$

You can estimate roots that are not perfect squares.

Example 5 Estimate Roots

Estimate each square root to the nearest whole number.

a. $\sqrt{15}$

Find the two perfect squares closest to 15. List some perfect squares.

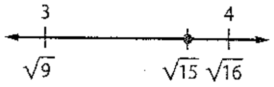
1, 4, 9, 16, 25, 36, …

15 is between 9 and 16.

$$9 < 15 < 16 \qquad \text{Write an inequality.}$$

$$\sqrt{9} < \sqrt{15} < \sqrt{16} \qquad \text{Take the square root of each number.}$$

$$3 < \sqrt{15} < 4 \qquad \text{Simplify.}$$



Since 15 is closer to 16 than 9, the best whole-number estimate for $\sqrt{15}$ is 4.

b. $\sqrt{130}$

Find the two perfect squares closest to 130. List some perfect squares.

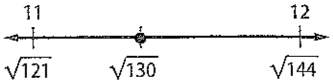
81, 100, 121, 144

130 is between 121 and 144.

$121 < 130 < 144$ Write an inequality.

$\sqrt{121} < \sqrt{130} < \sqrt{144}$ Take the square root of each number.

$11 < \sqrt{130} < 12$ Simplify.



StudyTip

Draw a Diagram
Graphing points on a number line can help you analyze your estimate for accuracy.

Since 130 is closer to 121 than to 144, the best whole number estimate for $\sqrt{130}$ is 11.

CHECK $\sqrt{130} \approx 11.4018$ Use a calculator.

Rounded to the nearest whole number, $\sqrt{130}$ is 11. So the estimate is valid.

Exercises

LESSON 0-3 Operations with Integers

Objective

- Add, subtract, multiply, and divide integers.

NewVocabulary

absolute value
opposites
additive inverses

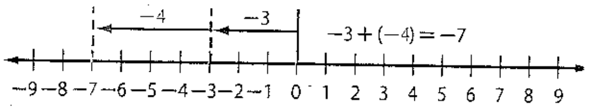
An integer is any number from the set {…, −3, −2, −1, 0, 1, 2, 3, …}. You can use a number line to add integers.

Example 1 Add Integers with the Same Sign

Use a number line to find −3 + (−4).

Step 1 Draw an arrow from 0 to −3.

Step 2 Draw a second arrow 4 units to the left to represent adding −4.



The second arrow ends at −7. So, −3 + (−4) = −7.

You can also use absolute value to add integers. The **absolute value** of a number is its distance from 0 on the number line.

Same Signs (+ + or − −)		Different Signs (+ − or − +)	
3 + 5 = 8	3 and 5 are positive. Their sum is positive.	3 + (−5) = −2	−5 has the greater absolute value. Their sum is negative.
−3 + (−5) = −8	−3 and −5 are negative. Their sum is negative.	−3 + 5 = 2	5 has the greater absolute value. Their sum is positive.

Example 2 Add Integers Using Absolute Value

Find −11 + (−7).

$$-11 + (-7) = -(|-11| + |-7|)$$
Add the absolute values. Both numbers are negative, so the sum is negative.

$$= -(11 + 7)$$
Absolute values of nonzero numbers are always positive.

$$= -18$$
Simplify.

Every positive integer can be paired with a negative integer. These pairs are called **opposites**. A number and its opposite are **additive inverses**. Additive inverses can

StudyTip

Products and Quotients The product or quotient of two numbers having the *same sign* is positive. The product or quotient of two numbers having *different signs* is negative.

Same Signs (+ + or − −)		Different Signs (+ − or − +)	
3(5) = 15	3 and 5 are positive. Their product is positive.	3(−5) = −15	3 and −5 have different signs. Their product is negative.
−3(−5) = 15	−3 and −5 are negative. Their product is positive.	−3(5) = −15	−3 and 5 have different signs. Their product is negative.

Example 4 Multiply and Divide Integers

PT

Find each product or quotient.

a. **4(−5)**

4(−5) = −20 different signs → negative product

b. **−51 ÷ (−3)**

−51 ÷ (−3) = 17 same sign → positive quotient

c. **−12(−14)**

−12(−14) = 168 same sign → positive product

d. **−63 ÷ 7**

−63 ÷ 7 = −9 different signs → negative quotient

LESSON 0-4 Adding and Subtracting Rational Numbers

Objective

- Compare and order; add and subtract rational numbers.

You can use different methods to compare rational numbers. One way is to compare two fractions with common denominators. Another way is to compare decimals.

PT

Example 1 Compare Rational Numbers

Replace ● with <, >, or = to make $\frac{2}{3}$ ● $\frac{5}{6}$ a true sentence.

Method 1 Write the fractions with the same denominator.

The least common denominator of $\frac{2}{3}$ and $\frac{5}{6}$ is 6.

$\frac{2}{3} = \frac{4}{6}$

$\frac{5}{6} = \frac{5}{6}$

Since $\frac{4}{6} < \frac{5}{6}$, $\frac{2}{3} < \frac{5}{6}$.

Method 2 Write as decimals.

Write $\frac{2}{3}$ and $\frac{5}{6}$ as decimals. You may want to use a calculator.

2 $\boxed{\div}$ 3 $\boxed{\text{ENTER}}$ **.6666666667**

So, $\frac{2}{3} = 0.\overline{6}$.

5 $\boxed{\div}$ 6 $\boxed{\text{ENTER}}$ **.8333333333**

So, $\frac{5}{6} = 0.8\overline{3}$.

Since $0.\overline{6} < 0.8\overline{3}$, $\frac{2}{3} < \frac{5}{6}$.

You can order rational numbers by writing all of the fractions as decimals.

PT

Example 2 Order Rational Numbers

Order $5\frac{2}{9}$, $5\frac{3}{8}$, 4.9, and $-5\frac{3}{5}$ from least to greatest.

$5\frac{2}{9} = 5.\overline{2}$ $\quad$ $5\frac{3}{8} = 5.375$

$4.9 = 4.9$ $\quad$ $-5\frac{3}{5} = -5.6$

$-5.6 < 4.9 < 5.\overline{2} < 5.375$. So, from least to greatest, the numbers are $-5\frac{3}{5}$, 4.9, $5\frac{2}{9}$, and $5\frac{3}{8}$.

To add or subtract fractions with the same denominator, add or subtract the numerators and write the sum or difference over the denominator.

Example 3 Add and Subtract Like Fractions

Find each sum or difference. Write in simplest form.

a. $\frac{3}{5}+\frac{1}{5}$

$\frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}$ The denominators are the same. Add the numerators.

$=\frac{4}{5}$ Simplify.

b. $\frac{7}{16}-\frac{1}{16}$

$\frac{7}{16}-\frac{1}{16}=\frac{7-1}{16}$ The denominators are the same. Subtract the numerators.

$=\frac{6}{16}$ Simplify.

$=\frac{3}{8}$ Rename the fraction.

c. $\frac{4}{9}-\frac{7}{9}$

$\frac{4}{9}-\frac{7}{9}=\frac{4-7}{9}$ The denominators are the same. Subtract the numerators.

$=-\frac{3}{9}$ Simplify.

$=-\frac{1}{3}$ Rename the fraction.

StudyTip

Mental Math If the denominators of the fractions are the same, you can use mental math to determine the sum or difference.

To add or subtract fractions with unlike denominators, first find the least common denominator (LCD). Rename each fraction with the LCD, and then add or subtract. Simplify if possible.

Example 4 Add and Subtract Unlike Fractions

Find each sum or difference. Write in simplest form.

a. $\frac{1}{2}+\frac{2}{3}$

$\frac{1}{2}+\frac{2}{3}=\frac{3}{6}+\frac{4}{6}$ The LCD for 2 and 3 is 6. Rename $\frac{1}{2}$ as $\frac{3}{6}$ and $\frac{2}{3}$ as $\frac{4}{6}$.

$=\frac{3+4}{6}$ Add the numerators.

$=\frac{7}{6}$ or $1\frac{1}{6}$ Simplify.

b. $\frac{3}{8}-\frac{1}{3}$

$\frac{3}{8}-\frac{1}{3}=\frac{9}{24}-\frac{8}{24}$ The LCD for 8 and 3 is 24. Rename $\frac{3}{8}$ as $\frac{9}{24}$ and $\frac{1}{3}$ as $\frac{8}{24}$.

$=\frac{9-8}{24}$ Subtract the numerators.

$=\frac{1}{24}$ Simplify.

c. $\frac{2}{5}-\frac{3}{4}$

$\frac{2}{5}-\frac{3}{4}=\frac{8}{20}-\frac{15}{20}$ The LCD for 5 and 4 is 20. Rename $\frac{2}{5}$ as $\frac{8}{20}$ and $\frac{3}{4}$ as $\frac{15}{20}$.

$=\frac{8-15}{20}$ Subtract the numerators.

$=-\frac{7}{20}$ Simplify.

StudyTip

Number Line To use a number line, put your pencil at the first number. If you are adding or subtracting a positive number, then move left to find the difference. To find the sum, move your pencil to the right.

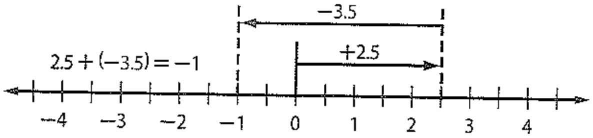
You can use a number line to add rational numbers.

Example 5 Add Decimals

Use a number line to find 2.5 + (−3.5).

Step 1 Draw an arrow from 0 to 2.5.

Step 2 Draw a second arrow 3.5 units to the left.



The second arrow ends at −1.

So, 2.5 + (−3.5) = −1.

You can also use absolute value to add rational numbers.

Same Signs (+ + or − −)		Different Signs (+ − or − +)	
3.1 + 2.5 = 5.6	3.1 and 2.5 are positive, so the sum is positive.	3.1 + (−2.5) = 0.6	3.1 has the greater absolute value, so the sum is positive.
−3.1 + (−2.5) = −5.6	−3.1 and −2.5 are negative, so the sum is negative.	−3.1 + 2.5 = −0.6	−3.1 has the greater absolute value, so the sum is negative.

Example 6 Use Absolute Value to Add Rational Numbers

Find each sum.

a. **−13.12 + (−8.6)**

$-13.12 + (-8.6) = -(|-13.12| + |-8.6|)$ Both numbers are negative, so the sum is negative.

$= -(13.12 + 8.6)$ Absolute values of nonzero numbers are always positive.

$= -21.72$ Simplify.

b. $\frac{7}{16} + \left(-\frac{3}{8}\right)$

$\frac{7}{16} + \left(-\frac{3}{8}\right) = \frac{7}{16} + \left(-\frac{6}{16}\right)$ The LCD is 16. Replace $-\frac{3}{8}$ with $-\frac{6}{16}$.

$= \left(\left|\frac{7}{16}\right| - \left|-\frac{6}{16}\right|\right)$ Subtract the absolute values. Because $\left|\frac{7}{16}\right|$ is greater than $\left|-\frac{6}{16}\right|$, the result is positive.

$= \frac{7}{16} - \frac{6}{16}$ Absolute values of nonzero numbers are always positive.

$= \frac{1}{16}$ Simplify.

To subtract a negative rational number, add its inverse.

Example 7 Subtract Decimals

Find $-32.25 - (-42.5)$.

$-32.25 - (-42.5) = -32.25 + 42.5$	To subtract -42.5, add its inverse.
$= \lvert 42.5 \rvert - \lvert -32.25 \rvert$	Subtract the absolute values. Because $\lvert 42.5 \rvert$ is greater than $\lvert -32.25 \rvert$, the result is positive.
$= 42.5 - 32.25$	Absolute values of nonzero numbers are always positive.
$= 10.25$	Simplify.

LESSON 0-5 Multiplying and Dividing Rational Numbers

Objective

- Multiply and divide rational numbers.

NewVocabulary

multiplicative inverses
reciprocals

The product or quotient of two rational numbers having the *same sign* is positive. The product or quotient of two rational numbers having *different signs* is negative.

Example 1 Multiply and Divide Decimals

Find each product or quotient.

a. $7.2(-0.2)$

different signs ⟶ negative product

$7.2(-0.2) = -1.44$

b. $-23.94 \div (-10.5)$

same sign ⟶ positive quotient

$-23.94 \div (-10.5) = 2.28$

To multiply fractions, multiply the numerators and multiply the denominators. If the numerators and denominators have common factors, you can simplify before you multiply by canceling.

Example 2 Multiply Fractions

Find each product.

a. $\frac{2}{5} \cdot \frac{1}{3}$

$\frac{2}{5} \cdot \frac{1}{3} = \frac{2 \cdot 1}{5 \cdot 3}$ Multiply the numerators. Multiply the denominators.

$= \frac{2}{15}$ Simplify.

b. $\frac{3}{5} \cdot 1\frac{1}{2}$

$\frac{3}{5} \cdot 1\frac{1}{2} = \frac{3}{5} \cdot \frac{3}{2}$ Write $1\frac{1}{2}$ as an improper fraction.

$= \frac{3 \cdot 3}{5 \cdot 2}$ Multiply the numerators. Multiply the denominators.

$= \frac{9}{10}$ Simplify.

c. $\frac{1}{4} \cdot \frac{2}{9}$

$\frac{1}{4} \cdot \frac{2}{9} = \frac{1}{\underset{2}{\cancel{4}}} \cdot \frac{\overset{1}{\cancel{2}}}{9}$ Divide by the GCF, 2.

$= \frac{1 \cdot 1}{2 \cdot 9}$ or $\frac{1}{18}$ Multiply the numerators. Multiply the denominators and simplify.

Example 3 Multiply Fractions with Different Signs

Find $\left(-\frac{3}{4}\right)\left(\frac{3}{8}\right)$.

$\left(-\frac{3}{4}\right)\left(\frac{3}{8}\right) = -\left(\frac{3}{4} \cdot \frac{3}{8}\right)$ different signs ⟶ negative product

$= -\left(\frac{3 \cdot 3}{4 \cdot 8}\right)$ or $\frac{9}{32}$ Multiply the numerators. Multiply the denominators and simplify.

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**.

Example 4 Find the Reciprocal

Name the reciprocal of each number.

a. $\frac{3}{8}$

$\frac{3}{8} \cdot \frac{8}{3} = 1$ The product is 1.

The reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$.

b. $2\frac{4}{5}$

$2\frac{4}{5} = \frac{14}{5}$ Write $2\frac{4}{5}$ as $\frac{14}{5}$.

$\frac{14}{5} \cdot \frac{5}{14} = 1$ The product is 1.

The reciprocal of $2\frac{4}{5}$ is $\frac{5}{14}$.

To divide one fraction by another fraction, multiply the dividend by the reciprocal of the divisor.

Example 5 Divide Fractions

Find each quotient.

a. $\frac{1}{3} \div \frac{1}{2}$

$\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{1}$ Multiply $\frac{1}{3}$ by $\frac{2}{1}$, the reciprocal of $\frac{1}{2}$.

$= \frac{2}{3}$ Simplify.

b. $\frac{3}{8} \div \frac{2}{3}$

$\frac{3}{8} \div \frac{2}{3} = \frac{3}{8} \cdot \frac{3}{2}$ Multiply $\frac{3}{8}$ by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.

$= \frac{9}{16}$ Simplify.

StudyTip

Use Estimation You can justify your answer by using estimation. $\frac{3}{8}$ is close to $\frac{1}{2}$ and $\frac{2}{3}$ is close to 1. So, the quotient is close to $\frac{1}{2}$ divided by 1 or $\frac{1}{2}$.

c. $\frac{3}{4} \div 2\frac{1}{2}$

$\frac{3}{4} \div 2\frac{1}{2} = \frac{3}{4} \div \frac{5}{2}$ Write $2\frac{1}{2}$ as an improper fraction

$= \frac{3}{4} \cdot \frac{2}{5}$ Multiply $\frac{3}{4}$ by $\frac{2}{5}$, the reciprocal of $2\frac{1}{2}$.

$= \frac{6}{20}$ or $\frac{3}{10}$ Simplify.

d. $-\frac{1}{5} \div \left(-\frac{3}{10}\right)$

$-\frac{1}{5} \div \left(-\frac{3}{10}\right) = -\frac{1}{5} \cdot \left(-\frac{10}{3}\right)$ Multiply $-\frac{1}{5}$ by $-\frac{10}{3}$, the reciprocal of $-\frac{3}{10}$.

$= \frac{10}{15}$ or $\frac{2}{3}$ Same sign ⟶ positive quotient; simplify.

LESSON

0-6 The Percent Proportion

Objective

- Use and apply the percent proportion.

NewVocabulary

percent
percent proportion

A **percent** is a ratio that compares a number to 100. To write a percent as a fraction, express the ratio as a fraction with a denominator of 100. Fractions should be expressed in simplest form.

Example 1 Percents as Fractions

Express each percent as a fraction or mixed number.

a. 79%

$79\% = \frac{79}{100}$ Definition of percent

b. 107%

$107\% = \frac{107}{100}$ Definition of percent

$= 1\frac{7}{100}$ Simplify.

c. 0.5%

$0.5\% = \frac{0.5}{100}$ Definition of percent

$= \frac{5}{1000}$ Multiply the numerator and denominator by 10 to eliminate the decimal.

$= \frac{1}{200}$ Simplify.

In the **percent proportion**, the ratio of a part of something to the whole (base) is equal to the percent written as a fraction.

$$\begin{array}{l} \text{part} \rightarrow \\ \text{whole} \rightarrow \end{array} \frac{a}{b} = \frac{p}{100} \leftarrow \text{percent}$$

$$\textbf{Example: } \overset{\text{percent}}{\overset{\downarrow}{25\%}} \text{ of } \overset{\text{whole}}{\overset{\downarrow}{40}} \text{ is } \overset{\text{part}}{\overset{\downarrow}{10.}}$$

You can use the percent proportion to find the part.

Example 2 Find the Part

40% of 30 is what number?

$\frac{a}{b} = \frac{p}{100}$ The percent is 40, and the base is 30. Let *a* represent the part.

$\frac{a}{30} = \frac{40}{100}$ Replace *b* with 30 and *p* with 40.

$100a = 30(40)$ Find the cross products.

$100a = 1200$ Simplify.

$\frac{100a}{100} = \frac{1200}{100}$ Divide each side by 100.

$a = 12$ Simplify.

The part is 12. So, 40% of 30 is 12.

You can also use the percent proportion to find the percent of the base.

Example 3 Find the Percent

SURVEYS Kelsey took a survey of students in her lunch period. 42 out of the 70 students Kelsey surveyed said their family had a pet. What percent of the students had pets?

$\frac{a}{b} = \frac{p}{100}$	The part is 42, and the base is 70. Let *p* represent the percent.
$\frac{42}{70} = \frac{p}{100}$	Replace *a* with 42 and *b* with 70.
$4200 = 70p$	Find the cross products.
$\frac{4200}{70} = \frac{70p}{70}$	Divide each side by 70.
$60 = p$	Simplify.

The percent is 60, so $\frac{60}{100}$ or 60% of the students had pets.

StudyTip

Percent Proportion In percent problems, the whole, or base usually follows the word *of*.

Example 4 Find the Whole

67.5 is 75% of what number?

$\frac{a}{b} = \frac{p}{100}$	The percent is 75, and the part is 67.5. Let *b* represent the base.
$\frac{67.5}{b} = \frac{75}{100}$	Replace *a* with 67.5 and *p* with 75.
$6750 = 75b$	Find the cross products.
$\frac{6750}{75} = \frac{75b}{75}$	Divide each side by 75.
$90 = b$	Simplify.

The base is 90, so 67.5 is 75% of 90.

LESSON 0-7 Perimeter

Objective

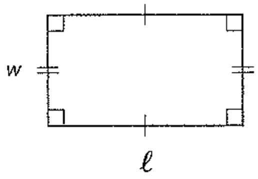
- Find the perimeter of two-dimensional figures.

NewVocabulary

perimeter
circle
diameter
circumference
center
radius

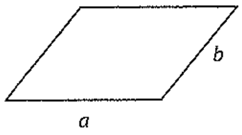
Perimeter is the distance around a figure. Perimeter is measured in linear units.

Rectangle



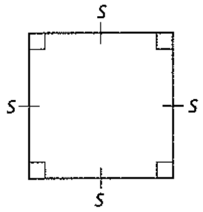
$P = 2(\ell + w)$ or
$P = 2\ell + 2w$

Parallelogram



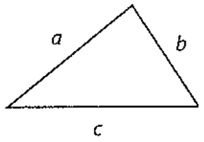
$P = 2(a + b)$ or
$P = 2a + 2b$

Square



$P = 4s$

Triangle

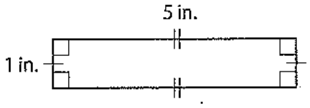


$P = a + b + c$

Example 1 Perimeters of Rectangles and Squares

Find the perimeter of each figure.

a. a rectangle with a length of 5 inches and a width of 1 inch



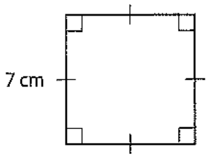
$P = 2(\ell + w)$ Perimeter formula

$= 2(5 + 1)$ $\ell = 5$, $w = 1$

$= 2(6)$ Add.

$= 12$ The perimeter is 12 inches.

b. a square with a side length of 7 centimeters



$P = 4s$ Perimeter formula

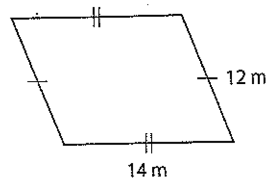
$= 4(7)$ Replace s with 7.

$= 28$ The perimeter is 28 centimeters.

Example 2 Perimeters of Parallelograms and Triangles

Find the perimeter of each figure.

a.



$P = 2(a + b)$ Perimeter formula

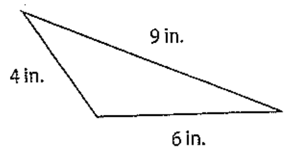
$= 2(14 + 12)$ $a = 14$, $b = 12$

$= 2(26)$ Add.

$= 52$ Multiply.

The perimeter of the parallelogram is 52 meters.

b.



$P = a + b + c$ Perimeter formula

$= 4 + 6 + 9$ $a = 4$, $b = 6$, $c = 9$

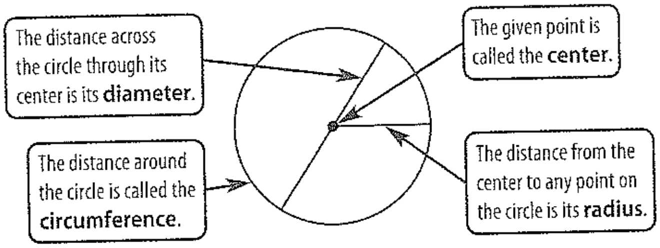
$= 19$ Add.

The perimeter of the triangle is 19 inches.

StudyTip

Congruent Marks The hash marks on the figures indicate sides that have the same length.

A **circle** is the set of all points in a plane that are the same distance from a given point.



The formula for the circumference of a circle is $C = \pi d$ or $C = 2\pi r$.

Example 3 Circumference

Find each circumference to the nearest tenth.

a. The radius is 4 feet.

$C = 2\pi r$ Circumference formula

$= 2\pi(4)$ Replace r with 4.

$= 8\pi$ Simplify.

The exact circumference is 8π feet.

8 [π] [ENTER] **25.13274123**

The circumference is about 25.1 feet.

b. The diameter is 15 centimeters.

$C = \pi d$ Circumference formula

$= \pi(15)$ Replace d with 15.

$= 15\pi$ Simplify.

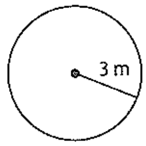
≈ 47.1 Use a calculator to evaluate 15π.

The circumference is about 47.1 centimeters.

StudyTip

Pi To perform a calculation that involves π, use a calculator.

c.



$C = 2\pi r$ Circumference formula

$= 2\pi(3)$ Replace r with 3.

$= 6\pi$ Simplify.

≈ 18.8 Use a calculator to evaluate 6π.

The circumference is about 18.8 meters.

LESSON 0-8 Area

Objective

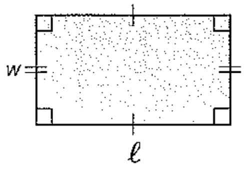
- Find the area of two-dimensional figures.

NewVocabulary

area

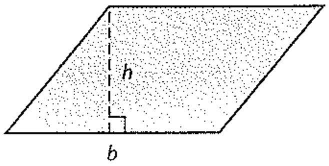
Area is the number of square units needed to cover a surface. Area is measured in square units.

Rectangle



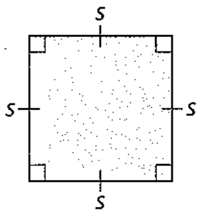
$A = \ell w$

Parallelogram



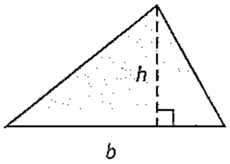
$A = bh$

Square



$A = s^2$

Triangle

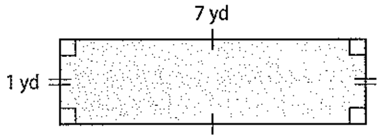


$A = \frac{1}{2}bh$

Example 1 Areas of Rectangles and Squares

Find the area of each figure.

a. a rectangle with a length of 7 yards and a width of 1 yard

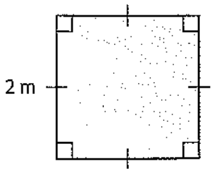


$A = \ell w$ Area formula

$= 7(1)$ $\ell = 7, w = 1$

$= 7$ The area of the rectangle is 7 square yards.

b. a square with a side length of 2 meters



$A = s^2$ Area formula

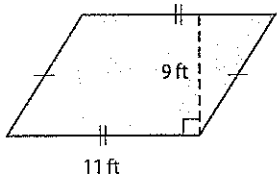
$= 2^2$ $s = 2$

$= 4$ The area is 4 square meters.

Example 2 Areas of Parallelograms and Triangles

Find the area of each figure.

a. **a parallelogram with a base of 11 feet and a height of 9 feet**



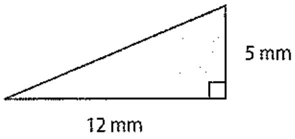
$A = bh$ Area formula

$= 11(9)$ $b = 11$, $h = 9$

$= 99$ Multiply.

The area is 99 square feet.

b. **a triangle with a base of 12 millimeters and a height of 5 millimeters**



$A = \frac{1}{2}bh$ Area formula

$= \frac{1}{2}(12)(5)$ $b = 12$, $h = 5$

$= 30$ Multiply.

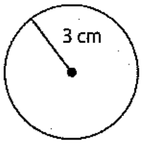
The area is 30 square millimeters.

The formula for the area of a circle is $A = \pi r^2$.

Example 3 Areas of Circles

Find the area of each circle to the nearest tenth.

a. **a radius of 3 centimeters**



$A = \pi r^2$ Area formula

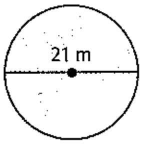
$= \pi(3)^2$ Replace r with 3.

$= 9\pi$ Simplify.

≈ 28.3 Use a calculator to evaluate 9π.

The area is about 28.3 square centimeters.

b. **a diameter of 21 meters**



$A = \pi r^2$ Area formula

$= \pi(10.5)^2$ Replace r with 10.5.

$= 110.25\pi$ Simplify.

≈ 346.4 Use a calculator to evaluate 110.25π.

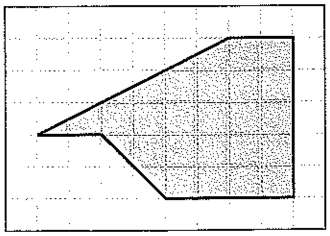
The area is about 346.4 square meters.

StudyTip

Mental Math You can use mental math to check your solutions. Square the radius and then multiply by 3.

Example 4 Estimate Area

Estimate the area of the polygon if each square represents 1 square mile.



One way to estimate the area is to count each square as one unit and each partial square as a half unit, no matter how large or small.

$A \approx$ squares + partial squares

$\approx 21(1) + 8(0.5)$ 21 whole squares and 8 partial squares

$\approx 21 + 4$ or 25

The area of the polygon is about 25 square miles.

LESSON 0-9 Volume

Objective

- Find the volumes of rectangular prisms and cylinders.

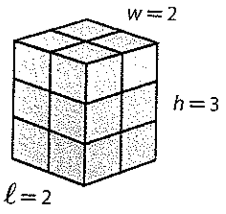
NewVocabulary

volume

Volume is the measure of space occupied by a solid. Volume is measured in cubic units.

To find the volume of a rectangular prism, multiply the length times the width times the height. The formula for the volume of a rectangular prism is shown below.

$$V = \ell \cdot w \cdot h$$



The prism at the right has a volume of $2 \cdot 2 \cdot 3$ or 12 cubic units.

Example 1 Volumes of Rectangular Prisms

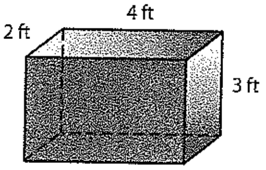
Find the volume of each rectangular prism.

a. The length is 8 centimeters, the width is 1 centimeter, and the height is 5 centimeters.

$V = \ell \cdot w \cdot h$	Volume formula
$= 8 \cdot 1 \cdot 5$	Replace ℓ with 8, w with 1, and h with 5.
$= 40$	Simplify.

The volume is 40 cubic centimeters.

b.



The prism has a length of 4 feet, width of 2 feet, and height of 3 feet.

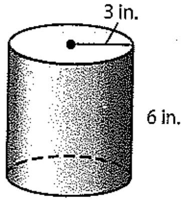
$V = \ell \cdot w \cdot h$	Volume formula
$= 4 \cdot 2 \cdot 3$	Replace ℓ with 4, w with 2, and h with 3.
$= 24$	Simplify.

The volume is 24 cubic feet.

The volume of a solid is the product of the area of the base and the height of the solid. For a cylinder, the area of the base is πr^2. So the volume is $V = \pi r^2 h$.

Example 2 Volume of a Cylinder

Find the volume of the cylinder.



$V = \pi r^2 h$	Volume of a cylinder
$= \pi(3^2)6$	$r = 3$, $h = 6$
$= 54\pi$	Simplify.
≈ 169.6	Use a calculator.

The volume is about 169.6 cubic inches.

LESSON 0-10 Surface Area

Objective

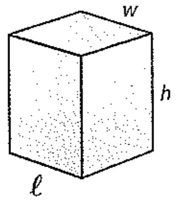
- Find the surface areas of rectangular prisms and cylinders.

NewVocabulary
surface area

Surface area is the sum of the areas of all the surfaces, or faces, of a solid. Surface area is measured in square units.

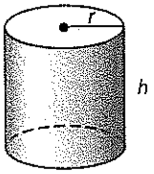
KeyConcept Surface Area

Prism



$S = 2\ell w + 2\ell h + 2wh$

Cylinder

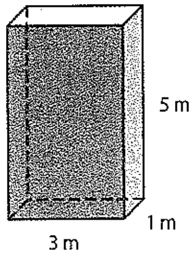


$S = 2\pi rh + 2\pi r^2$

Example 1 Find Surface Areas

Find the surface area of each solid. Round to the nearest tenth if necessary.

a.

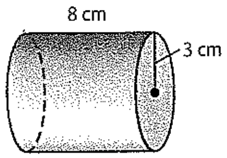


The prism has a length of 3 meters, width of 1 meter, and height of 5 meters.

$S = 2\ell w + 2\ell h + 2wh$	Surface area formula
$= 2(3)(1) + 2(3)(5) + 2(1)(5)$	$\ell = 3, w = 1, h = 5$
$= 6 + 30 + 10$	Multiply.
$= 46$	Add.

The surface area is 46 square meters.

b.



The height is 8 centimeters and the radius of the base is 3 centimeters. The surface area is the sum of the area of each base, $2\pi r^2$, and the area of the side, given by the circumference of the base times the height or $2\pi rh$.

$S = 2\pi rh + 2\pi r^2$	Formula for surface area of a cylinder.
$= 2\pi(3)(8) + 2\pi(3^2)$	$r = 3, h = 8$
$= 48\pi + 18\pi$	Simplify.
$\approx 207.3 \text{ cm}^2$	Use a calculator.

LESSON 0-11

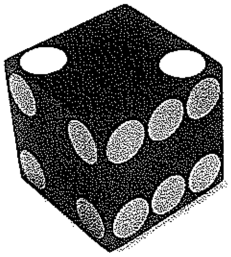
Simple Probability and Odds

Objective

- Find the probability and odds of simple events.

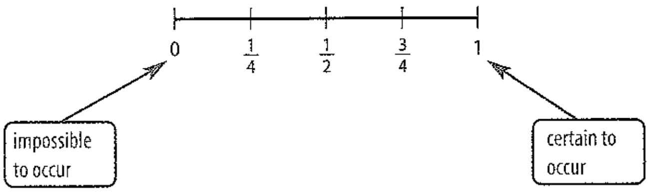
NewVocabulary

probability
sample space
equally likely
complements
tree diagram
odds



The **probability** of an event is the ratio of the number of favorable outcomes for the event to the total number of possible outcomes. When you roll a die, there are six possible outcomes: 1, 2, 3, 4, 5, or 6. This list of all possible outcomes is called the **sample space.**

When there are n outcomes and the probability of each one is $\frac{1}{n}$, we say that the outcomes are **equally likely.** For example, when you roll a die, the 6 possible outcomes are equally likely because each outcome has a probability of $\frac{1}{6}$. The probability of an event is always between 0 and 1, inclusive. The closer a probability is to 1, the more likely it is to occur.



Example 1 Find Probabilities

A die is rolled. Find each probability.

a. rolling a 1 or 5

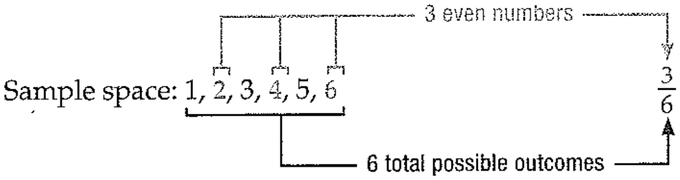
There are six possible outcomes. There are two favorable outcomes, 1 and 5.

$$\text{probability} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}} = \frac{2}{6}$$

So, $P(1 \text{ or } 5) = \frac{2}{6}$ or $\frac{1}{3}$.

b. rolling an even number

Three of the six outcomes are even numbers. So, there are three favorable outcomes.



So, $P(\text{even number}) = \frac{3}{6}$ or $\frac{1}{2}$.

The events for rolling a 1 and for *not* rolling a 1 are called **complements**.

$$\overset{P(1)}{\frac{1}{6}} + \overset{P(\text{not } 1)}{\frac{5}{6}} = \overset{P(\text{sum of probabilities})}{\frac{6}{6}} \text{ or } 1$$

The sum of the probabilities for any two complementary events is always 1.

Stockbyte/Getty Images

Example 2 Find Probabilities

A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find each probability.

a. blue

There are 7 blue chips and 28 total chips.

$$P(\text{blue chip}) = \frac{7}{28} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes}\\ \leftarrow \text{number of possible outcomes}\end{matrix}$$

$$= \frac{1}{4}$$

The probability can be stated as $\frac{1}{4}$, 0.25, or 25%.

b. red or yellow

There are 5 + 6 or 11 chips that are red or yellow.

$$P(\text{red or yellow}) = \frac{11}{28} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes}\\ \leftarrow \text{number of possible outcomes}\end{matrix}$$

$$\approx 0.39$$

The probability can be stated as $\frac{11}{28}$, about 0.39, or about 39%.

c. not green

There are 5 + 7 + 6 or 18 chips that are not green.

$$P(\text{not green}) = \frac{18}{28} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes}\\ \leftarrow \text{number of possible outcomes}\end{matrix}$$

$$= \frac{9}{14} \text{ or about } 0.64$$

The probability can be stated as $\frac{9}{14}$, about 0.64, or about 64%.

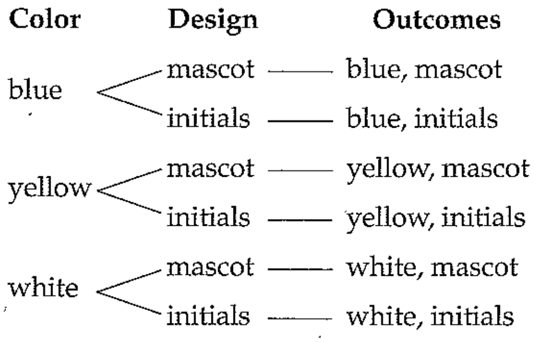
StudyTip

Alternate Method A chip drawn will either be green or not green. So, another method for finding *P*(not green) is to find *P*(green) and subtract that probability from 1.

One method used for counting the number of possible outcomes is to draw a **tree diagram**. The last column of a tree diagram shows all of the possible outcomes.

Example 3 Use a Tree Diagram to Count Outcomes

School baseball caps come in blue, yellow, or white. The caps have either the school mascot or the school's initials. Use a tree diagram to determine the number of different caps possible.



The tree diagram shows that there are 6 different caps possible.

StudyTip

Counting Outcomes When counting possible outcomes, make a column in your tree diagram for each part of the event.

This example is an illustration of the **Fundamental Counting Principle**, which relates the number of outcomes to the number of choices.

KeyConcept Fundamental Counting Principle

Words	If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by N can occur in $m \cdot n$ ways.
Example	If there are 4 possible sizes for fish tanks and 3 possible shapes, then there are $4 \cdot 3$ or 12 possible fish tanks.

Example 4 Use the Fundamental Counting Principle

a. An ice cream shop offers one, two, or three scoops of ice cream from among 12 different flavors. The ice cream can be served in a wafer cone, a sugar cone, or in a cup. Use the Fundamental Counting Principle to determine the number of choices possible.

There are 3 ways the ice cream is served, 3 different servings, and there are 12 different flavors of ice cream.

Use the Fundamental Counting Principle to find the number of possible choices.

number of scoops		number of flavors		number of serving options		number of choices of ordering ice cream
3	•	12	•	3	=	108

So, there are 108 different ways to order ice cream.

b. Jimmy needs to make a 3-digit password for his log-on name on a Web site. The password can include any digit from 0-9, but the digits may not repeat. How many possible 3-digit passwords are there?

If the first digit is a 4, then the next digit cannot be a 4.

We can use the Fundamental Counting Principle to find the number of possible passwords.

1st digit		2nd digit		3rd digit		number of passwords
10	•	9	•	8	=	720

So, there are 720 possible 3-digit passwords.

The **odds** of an event occurring is the ratio that compares the number of ways an event can occur (successes) to the number of ways it cannot occur (failures).

StudyTip

Odds The sum of the number of successes and the number of failures equals the size of the sample space, or the number of possible outcomes.

Example 5 Find the Odds

Find the odds of rolling a number less than 3.

There are six possible outcomes; 2 are successes and 4 are failures.

So, the odds of rolling a number less than 3 are $\frac{1}{2}$ or 1:2.

LESSON 0-12

Measures of Center, Variation, and Position

Objective

- Find measures of central tendency, variation, and position.

NewVocabulary

variable
data
measurement or quantitative data
categorical or qualitative data
univariate data
measures of center or central tendency
mean
median
mode
measures of spread or variation
range
quartile
measures of position
lower quartile
upper quartile
five-number summary
interquartile range
outlier

A **variable** is a characteristic of a group of people or objects that can assume different values called **data**. Data that have units and can be measured are called **measurement** or **quantitative data**. Data that can be organized into different categories are called **categorical** or **qualitative data**. Some examples of both types of data are listed below.

Measurement Data	Categorical Data
Times: 15 s, 20 s, 45 s, 19 s	Favorite color: blue, red, purple, green
Ages: 10 yr, 15 yr, 14 yr, 16 yr	Hair color: black, blonde, brown
Distance: 5 mi, 30 mi, 18 mi	Phone Numbers: 555-1234, 555-5678

Measurement data in one variable, called **univariate data**, are often summarized using a single number to represent what is average or typical. Measures of what is average are also called **measures of center** or **central tendency**. The most common measures of center are mean, median, and mode.

KeyConcept Measures of Center

- The **mean** is the sum of the values in a data set divided by the total number of values in the set.
- The **median** is the middle value or the mean of the two middle values in a set of data when the data are arranged in numerical order.
- The **mode** is the value or values that appear most often in a set of data. A set of data can have no mode, one mode, or more than one mode.

Example 1 Measures of Center

PT

BASEBALL The table shows the number of hits Marcus made for his team. Find the mean, median, and mode.

Team Played	Hits
Badgers	3
Hornets	6
Bulldogs	5
Vikings	2
Rangers	3
Panthers	7

Mean: To find the mean, find the sum of all the hits and divide by the number of games in which he made these hits.

$$\text{mean} = \frac{3+6+5+2+3+7}{6} = \frac{26}{6} \text{ or about 4 hits}$$

Median: To find the median, order the numbers from least to greatest and find the middle value or values.

2, 3, 3, 5, 6, 7

$$\frac{3+5}{2} \text{ or 4 hits}$$

Since there is an even number of values, find the mean of the middle two.

Mode: From the arrangement of the data values, we can see that the value that occurs most often in the set is 3, so the mode of the data set is 3 hits.

Marcus's mean and median number of hits for these games was 4, and his mode was 3 hits.

Two very different data sets can have the same mean, so statisticians also use **measures of spread** or **variation** to describe how widely the data values vary. One such measure is the **range**, which is the difference between the greatest and least values in a set of data.

Example 2 Range

WALKING The times in minutes it took Olivia to walk to school each day this week are 18, 15, 15, 12, and 14. Find the range.

range = greatest value − least value   Definition of range

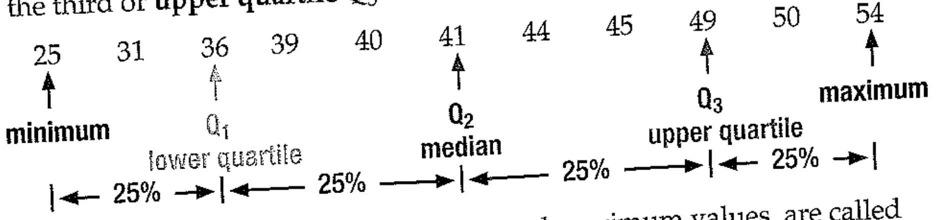
= 18 − 12 or 6   The greatest value is 18, and the least value is 12.

The range of the times is 6 minutes.

Statisticians often talk about the position of a value relative to other values in a set. **Quartiles** are common **measures of position** that divide a data set arranged in ascending order into four groups, each containing about one fourth or 25% of the data. The median marks the second quartile Q_2 and separates the data into upper and lower halves. The first or **lower quartile** Q_1 is the median of the lower half, while the third or **upper quartile** Q_3 is the median of the upper half.

StudyTip

Calculating Quartiles When the number of values in a set of data is odd, the median is not included in either half of the data when calculating Q_1 or Q_3.

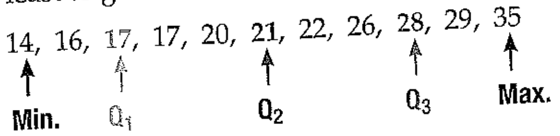


The three quartiles, along with the minimum and maximum values, are called a **five- number summary** of a data set.

Example 3 Five-Number Summary

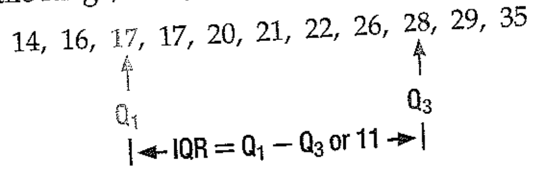
FUNDRAISER The number of boxes of donuts Aang sold for a fundraiser each day for the last 11 days were 22, 16, 35, 26, 14, 17, 28, 29, 21, 17, and 20. Find the minimum, lower quartile, median, upper quartile, and maximum of the data set. Then interpret this five-number summary.

Order the data from least to greatest. Use the list to determine the quartiles.



The minimum is 14, the lower quartile is 17, the median is 21, the upper quartile is 28, and the maximum is 35. Over the last 11 days, Aang sold a minimum of 14 boxes and a maximum of 35 boxes. He sold fewer than 17 boxes 25% of the time, fewer than 21 boxes 50% of the time, and fewer than 28 boxes 75% of the time.

The difference between the upper and lower quartiles is called the **interquartile range**. The interquartile range, or IQR, contains about 50% of the values.



Before deciding on which measure of center best describes a data set, check for outliers. An **outlier** is an extremely high or extremely low value when compared with the rest of the values in the set. To check for outliers, look for data values that are beyond the upper or lower quartiles by more than 1.5 times the interquartile range.

Example 4 Effect of Outliers

TEST SCORES Students taking a make-up test received the following scores: 88, 79, 94, 90, 45, 71, 82, and 88.

a. Identify any outliers in the data.

First determine the median and upper and lower quartiles of the data.

$$\begin{array}{ccccccccccc} 45, & 71, & & 79, & 82, & & 88, & 88, & & 90, & 94 \\ & & \uparrow & & & \uparrow & & & \uparrow & & \\ & & Q_1 = \dfrac{71+79}{2} \text{ or } 75 & & & Q_2 = \dfrac{82+88}{2} \text{ or } 85 & & & Q_3 = \dfrac{88+90}{2} \text{ or } 89 & & \end{array}$$

StudyTip

Interquartile Range When the interquartile range is a small value, the data in the set are close together. A large interquartile range means that the data are spread out.

Find the interquartile range.

$IQR = Q_3 - Q_1 = 89 - 75$ or 14

Use the interquartile range to find the values beyond which any outliers would lie.

$Q_1 - 1.5(IQR)$	and	$Q_3 + 1.5(IQR)$	Values beyond which outliers lie
$75 - 1.5(14)$		$89 + 1.5(14)$	$Q_1 = 75$, $Q_3 = 89$, and $IQR = 14$
54		110	Simplify.

There are no scores greater than 110, but there is one score less than 54. The score of 45 can be considered an outlier for this data set.

b. Find the mean and median of the data set with and without the outlier. Describe what happens.

Data Set	Mean	Median
with outlier	$\dfrac{88+79+94+90+45+71+82+88}{8}$ or about 79.6	85
without outlier	$\dfrac{88+79+94+90+71+82+88}{7}$ or about 84.6	88

Removal of the outlier causes the mean and median to increase, but notice that the mean is affected more by the removal of the outlier than the median.

LESSON 0-13 Representing Data

:: Objective

- Represent sets of data using different visual displays.

NewVocabulary

frequency table
bar graph
cumulative frequency
histogram
line graph
stem-and-leaf plot
circle graph
box-and-whisker plot

Common Core State Standards

Content Standards

S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

A **frequency table** uses tally marks to record and display frequencies of events. A **bar graph** compares categories of data with bars representing the frequencies.

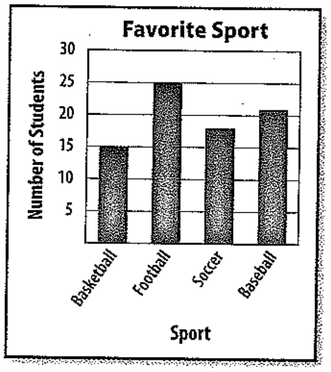
Example 1 Make a Bar Graph

Make a bar graph to display the data.

Sport	Tally	Frequency
basketball	𝍸 𝍸 𝍸	15
football	𝍸 𝍸 𝍸 𝍸 𝍸	25
soccer	𝍸 𝍸 𝍸 |||	18
baseball	𝍸 𝍸 𝍸 𝍸 |	21

Step 1 Draw a horizontal axis and a vertical axis. Label the axes as shown. Add a title.

Step 2 Draw a bar to represent each sport. The vertical scale is the number of students who chose each sport. The horizontal scale identifies the sport.



The **cumulative frequency** for each event is the sum of its frequency and the frequencies of all preceding events. A **histogram** is a type of bar graph used to display numerical data that have been organized into equal intervals.

Example 2 Make a Histogram and a Cumulative Frequency Histogram

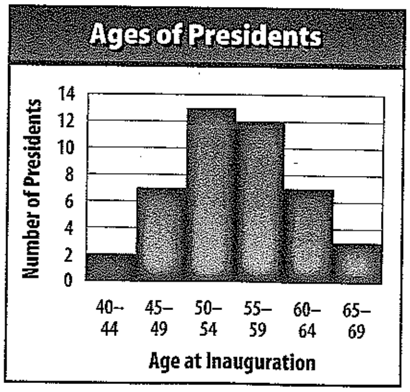
Make histograms of the frequency and the cumulative frequency.

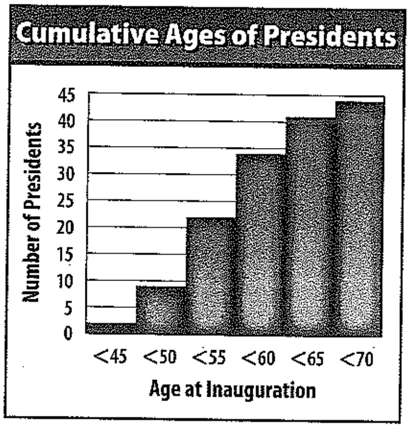
Age at Inauguration	40–44	45–49	50–54	55–59	60–64	65–69
U.S. Presidents	2	7	13	12	7	3

Find the cumulative frequency for each interval.

Age	< 45	< 50	< 55	< 60	< 65	< 70
Presidents	2	2 + 7 = 9	9 + 13 = 22	22 + 12 = 34	34 + 7 = 41	41 + 3 = 44

Make each histogram like a bar graph but with no space between the bars.





Another way to represent data is by using a line graph. A **line graph** usually shows how data change over a period of time.

Example 3 Make a Line Graph

Sales at the Marshall High School Store are shown in the table. Make a line graph of the data.

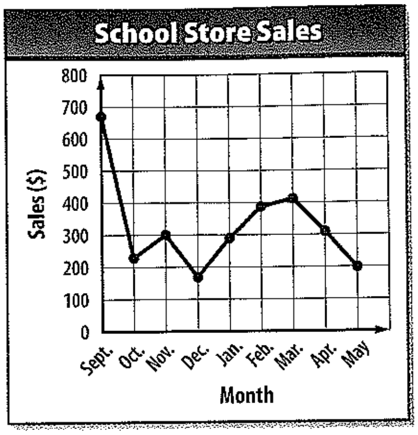
School Store Sales Amounts

September	$670	**December**	$168	**March**	$412
October	$229	**January**	$290	**April**	$309
November	$300	**February**	$388	**May**	$198

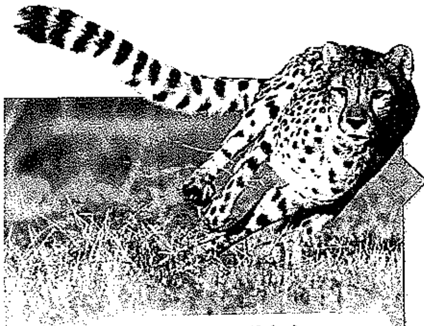
Step 1 Draw a horizontal axis and a vertical axis and label them as shown. Include a title.

Step 2 Plot the points.

Step 3 Draw a line connecting each pair of consecutive points.



Data can also be organized and displayed by using a stem-and-leaf plot. In a **stem-and-leaf plot**, the digits of the least place value usually form the *leaves*, and the rest of the digits form the *stems*.



Real-WorldLink

The fastest animal on land is the cheetah. Cheetahs can run at speeds up to 60 miles per hour.

Source: Infoplease

Real-World Example 4 Make a Stem-and-Leaf Plot

ANIMALS The speeds (mph) of 20 of the fastest land animals are listed at the right. Use the data to make a stem-and-leaf plot.

42	40	40	35	50
32	50	36	50	40
45	70	43	45	32
40	35	61	48	35

Source: *The World Almanac*

The least place value is ones. So, 32 miles per hour would have a stem of 3 and a leaf of 2.

Stem	Leaf
3	2 2 5 5 5 6
4	0 0 0 0 2 3 5 5 8
5	0 0 0
6	1
7	0

Key: 3|2 = 32

A **circle graph** is a graph that shows the relationship between parts of the data and the whole. The circle represents all of the data.

Example 5 Make a Circle Graph

The table shows how Lily spent 8 hours of one day at summer camp. Make a circle graph of the data.

Summer Camp	
Activity	**Hours**
canoeing	3
crafts	1
eating	2
hiking	2

First, find the ratio that compares the number of hours for each activity to 8. Then multiply each ratio by 360° to find the number of degrees for each section of the graph.

Canoeing: $\frac{3}{8} \cdot 360° = 135°$

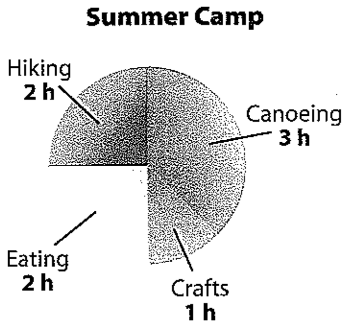
Crafts: $\frac{1}{8} \cdot 360° = 45°$

Eating: $\frac{2}{8} \cdot 360° = 90°$

Hiking: $\frac{2}{8} \cdot 360° = 90°$

WatchOut!

Circle Graphs The sum of the measures of each section of a circle graph should be 360°.



A **box-and-whisker plot** is a graphical representation of the five-number summary of a data set. The box in a box-and-whisker plot represents the interquartile range.

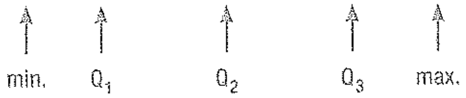
Example 6 Make a Box-and-Whisker Plot

Draw a box-and-whisker plot for these data. Describe how the outlier affects the quartile points.

14, 30, 16, 20, 18, 16, 20, 18, 22, 13, 8

Step 1 Order the data from least to greatest. Then determine the maximum, minimum and the quartiles.

8, 13, **14**, 16, 16, **18**, 18, 20, **20**, 22, 30



Determine the interquartile range.

$\text{IQR} = Q_3 - Q_1$

$= 20 - 14 \text{ or } 6$

Check to see if there are any outliers.

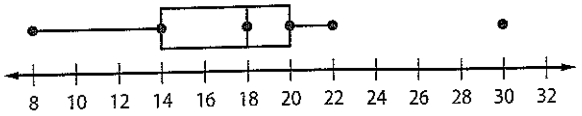
$14 - 1.5(6) = 5 \qquad 20 + 1.5(6) = 29$

Numbers less than 5 or greater than 29 are outliers.

The only outlier is 30.

Step 2 Draw a number line that includes the minimum and maximum values in the data. Place dots above the number line to represent the three quartile points, any outliers, the minimum value that is not an outlier, and the maximum value that is not an outlier.

Step 3 Draw the box and the whiskers. The vertical rules go through the quartiles. The outliers are not connected to the whiskers.



Step 4 Omit 30 from the data. Repeat Step 1 to determine Q_1, Q_2, and Q_3.

8, 13, **14,** 16, **16, 18,** 18, **20,** 20, 22

Q_1 $\quad Q_2 = 17$ $\quad Q_3$

Removing the outlier does not affect Q_1 or Q_2 and thus does not affect the interquartile range. The value of Q_2 changes from 18 to 17.

StudyTip

Parallel Box-and-Whisker Plot A double box-and-whisker plot is sometimes called a *parallel box-and-whisker plot.*

Example 7 Compare Data

CLIMATE Lucas is going to go to college in either Dallas or Nashville. He wants to live in a place that does not get too cold. So he decides to compare the average monthly low temperatures of each city.

Average Monthly Low Temperatures (°F)		
Month	**Dallas**	**Nashville**
Jan.	36	28
Feb.	41	31
Mar.	49	39
Apr.	56	47
May	65	57
June	73	65
July	77	70
Aug.	76	68
Sept.	69	61
Oct.	58	49
Nov.	47	40
Dec.	39	32

Source: weather.com

a. Draw a double box-and-whisker plot for the data.

Determine the quartiles and outliers for each city.

Dallas

36, 39, 41, 47, 49, 56, 58, 65, 69, 73, 76, 77

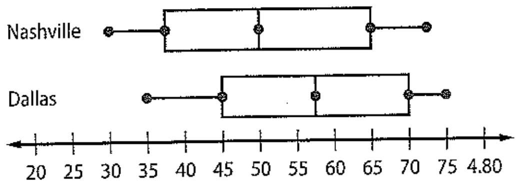
$Q_1 = 44$ $\quad Q_2 = 57$ $\quad Q_3 = 71$

Nashville

28, 31, 32, 39, 40, 47, 49, 57, 61, 65, 68, 70

$Q_1 = 35.5$ $\quad Q_2 = 48$ $\quad Q_3 = 63$

There are no outliers. Draw the plots using the same number line.



b. Use the double box-and-whisker plot to compare the data.

The interquartile range of temperatures for both cities is about the same. However, all quartiles of the Dallas temperatures are shifted to the right of those of Nashville, meaning Dallas has higher average low temperatures.

c. One night in August, a weather reporter stated the low for Nashville as being "only 65." Is it appropriate for the weather reporter to use the word only in the statement? Is 65 an unusually low temperature for Nashville in August? Explain your answer.

No, 65 is not an unusually low temperature for August in Nashville. It is lower than the average, but not by much.