Dear Enriched Algebra II Students,

Welcome to Enriched Algebra II. This advanced course will begin using prior knowledge from Algebra I at a fast pace. Please take time this summer to complete the attached review sheets to help sharpen your Algebra I skills. Many of you may still have your Algebra I notebook, which will be a great resource! There are many concepts that you are expected to know, as there is not time in class to go over review material. You will be given a pre-test at the beginning of the year to help assess the level you are entering Enriched Algebra II. A graphing calculator is not required this year; the TI 30 scientific calculator from previous years is recommended. I am looking forward to meeting you and having a wonderful 2018-19 school year!

Have a great summer!

Mrs. Benson

NAME ______________________ DATE ______________ PERIOD ________

Study Guide *Solving $x^2 + bx + c = 0$*

1

Factor $x^2 + bx + c$ To factor a trinomial of the form $x^2 + bx + c$, find two integers, m and p, whose sum is equal to b and whose product is equal to c.

Factoring $x^2 + bx + c$	$x^2 + bx + c = (x + m)(x + p)$, where $m + p = b$ and $mp = c$

Example 1: Factor each polynomial.

a. $x^2 + 7x + 10$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \_ 5 = 10$, let $m = 2$ and $p = 5$.

$x^2 + 7x + 10 = (x + 5)(x + 2)$

b. $x^2 - 8x + 7$

In this trinomial, $b = -8$ and $c = 7$. Notice that $m + p$ is negative and mp is positive, so m and p are both negative.

Since $-7 + (-1) = -8$ and $(-7)(-1) = 7$, $m = -7$ and $p = -1$.

$x^2 - 8x + 7 = (x - 7)(x - 1)$

Example 2 : Factor $x^2 + 6x - 16$.

In this trinomial, $b = 6$ and $c = -16$. This means $m + p$ is positive and mp is negative. Make a list of the factors of -16, where one factor of each pair is positive.

Factors of −16	Sum of Factors
1, −16	−15
−1, 16	15
2, −8	−6
−2, 8	6

Therefore, $m = -2$ and $p = 8$.

$x^2 + 6x - 16 = (x - 2)(x + 8)$

Exercises

Factor each polynomial.

1. $x^2 + 4x + 3$

3. $x^2 - 4x - 21$

4. $t^2 - 4t - 12$

5. $x^2 + 6x + 5$

6. $x^2 - 2x - 3$

7. $x^2 + 12x + 20$

8. $x^2 + 2xy + y^2$

NAME ______________________ DATE ______________________ PERIOD ________

Study Guide *Solving $ax^2 + bx + c = 0$*

2

Factor $ax^2 + bx + c$ To factor a trinomial of the form $ax^2 + bx + c$, find two integers, m and p whose product is equal to ac and whose sum is equal to b. If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

Example 1: Factor $2x^2 + 15x + 18$.
In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers that have a sum of 15 and a product of $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors with a sum of 15.

Factors of 36	Sum of Factors
1, 36	37
2, 18	20
3, 12	15

Use the pattern $ax^2 + mx + px + c$, with $a = 2$, $m = 3$, $p = 12$, and $c = 18$.

$$\begin{aligned} 2x^2 + 15x + 18 &= 2x^2 + 3x + 12x + 18 \\ &= (2x^2 + 3x) + (12x + 18) \\ &= x(2x + 3) + 6(2x + 3) \\ &= (x + 6)(2x + 3) \end{aligned}$$

Therefore, $2x^2 + 15x + 18 = (x + 6)(2x + 3)$.

Example 2: Factor $3x^2 - 3x - 18$.
Note that the GCF of the terms $3x^2$, $3x$, and 18 is 3. First factor out this GCF. $3x^2 - 3x - 18 = 3(x^2 - x - 6)$. Now factor $x^2 - x - 6$. Since $a = 1$, find the two factors of -6 with a sum of -1.

Factors of −6	Sum of Factors
1, −6	−5
−1, 6	5
−2, 3	1
2, −3	−1

Now use the pattern $(x + m)(x + p)$ with $m = 2$ and $p = -3$.

$x^2 - x - 6 = (x + 2)(x - 3)$

The complete factorization is
$3x^2 - 3x - 18 = 3(x + 2)(x - 3)$.

Exercises

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2x^2 - 3x - 2$

2. $3m^2 - 8m - 3$

3. $16r^2 - 8r + 1$

4. $6x^2 - 7x + 18$

5. $18 + 11y + 2y^2$

NAME ______________________________ DATE ____________________ PERIOD __________

3

Study Guide *Solving $ax^2 + bx + c = 0$ (continued)*

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve some equations of the form $ax^2 + bx + c = 0$.

Example: Solve $12x^2 + 3x = 2 - 2x$. Check your solutions.

$12x^2 + 3x = 2 - 2x$	Original equation
$12x^2 + 5x - 2 = 0$	Rewrite equation so that one side equals 0.
$(3x + 2)(4x - 1) = 0$	Factor the left side.
$3x + 2 = 0$ or $4x - 1 = 0$	Zero Product Property
$x = -\frac{2}{3}$ $\quad$ $x = \frac{1}{4}$	Solve each equation.

The solution set is $\left\{-\frac{2}{3}, \frac{1}{4}\right\}$.

Since $12\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) = 2 - 2\left(-\frac{2}{3}\right)$ and $12\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) = 2 - 2\left(\frac{1}{4}\right)$, the solutions check.

Exercises

Solve each equation. Check the solutions.

1. $8x^2 + 2x - 3 = 0$

2. $3n^2 - 2n - 5 = 0$

3. $2k^2 - 40 = -11k$

4. $-7 - 18x + 9x^2 = 0$

5. $8x^2 + 5x = 3 + 7x$

6. $4a^2 - 18a + 5 = 15$

7. $3b^2 - 18b = 10b - 49$

8. The difference of the squares of two consecutive odd integers is 24. Find the integers.

9. **GEOMETRY** The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width. The area is 300 square yards. What are the dimensions?

NAME ______________________ DATE ______________ PERIOD ________

4

Study Guide *Multiplication Properties of Exponents*

Multiply Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1: Simplify $(3x^6)(5x^2)$.

$(3x^6)(5x^2) = (3)(5)(x^6 \cdot x^2)$ Group the coefficients and the variables

$= (3 \cdot 5)(x^{6+2})$ Product of Powers

$= 15x^8$ Simplify.

The product is $15x^8$.

Example 2: Simplify $(-4a^3b)(3a^2b^5)$.

$(-4a^3b)(3a^2b^5) = (-4)(3)(a^3 \cdot a^2)(b \cdot b^5)$

$= -12(a^{3+2})(b^{1+5})$

$= -12a^5b^6$

The product is $-12a^5b^6$.

Simplify Expressions An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and any integers m and p, $(a^m)^p = a^{mp}$.
Power of a Product	For any numbers a and b and any integer m, $(ab)^m = a^mb^m$.

We can combine and use these properties to simplify expressions involving monomials.

Example: Simplify $(-2ab^2)^3(a^2)^4$.

$(-2ab^2)^3(a^2)^4 = (-2ab^2)^3(a^8)$ Power of a Power

$= (-2)^3(a^3)(b^2)^3(a^8)$ Power of a Product

$= (-2)^3(a^3)(a^8)(b^2)^3$ Group the coefficients and the variables

$= (-2)^3(a^{11})(b^2)^3$ Product of Powers

$= -8a^{11}b^6$ Power of a Power

The product is $-8a^{11}b^6$.

Exercises. Simplify each expression.

1. $x(x^2)(x^4)$
2. $m \cdot m^5$
3. $(-x^3)(-x^4)$
4. $\frac{1}{3}(2a^3b)(6b^3)$
5. $(-4x^3)(-5x^7)$
6. $(-3j^2k^4)(2jk^6)$
7. $(-5xy)(4x^2)(y^4)$
8. $(10x^3yz^2)(-2xy^5z)$
9. $(y^5)^2$
10. $(2a^3b^2)(b^3)^2$
11. $(-4xy)^3(-2x^2)^3$
12. $(-3j^2k^3)^2(2j^2k)^3$
13. $(25a^2b)^3\left(\frac{1}{5}abf\right)^2$
14. $(2xy)^2(-3x^2)(4y^4)$
15. $(2x^3y^2z^2)^3(x^2z)^4$

NAME ______________________ DATE ______________________ PERIOD __________

Study Guide *Division Properties of Exponents*

5

Divide Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers *m* and *n* and any nonzero number *a*, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer *m* and any real numbers *a* and *b*, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{a^m}$.

Example1: Simplify $\frac{a^4b^7}{ab^2}$. Assume that no denominator equals zero.

$\frac{a^4b^7}{ab^2} = \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right)$ Group powers with the same base.

$= (a^{4-1})(b^{7-2})$ Quotient of Powers

$= a^3b^5$ Simplify.

The quotient is a^3b^5

Example 2: Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that no denominator equals zero.

$\left(\frac{2a^3b^5}{3b^2}\right)^3 = \frac{(2a^3b^5)^3}{(3b^2)^3}$ Power of a Quotient

$= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3}$ Power of a Product

$= \frac{8a^9b^{15}}{27b^6}$ Power of a Power

$= \frac{8a^9b^9}{27}$ Quotient of Powers

The quotient is $\frac{8a^9b^9}{27}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{5^5}{5^2}$

2. $\frac{m^6}{m^4}$

3. $\frac{p^5n^4}{p^2n}$

4. $\frac{xy^6}{y^4x}$

5. $\left(\frac{2a^2b}{a}\right)^3$

6. $\left(\frac{4p^4r^4}{3p^2r^2}\right)^3$

7. $\left(\frac{2r^5w^3}{r^4w^3}\right)^4$

8. $\left(\frac{3r^6n^3}{2r^5n}\right)^4$

9. $\frac{r^7n^7t^2}{n^3r^3t^2}$

Study Guide *Division Properties of Exponents* *(continued)*

6

Division Properties of Exponents

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents..

Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}\,n$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example: Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that no denominator equals zero.

$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$ Group powers with the same base.

$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$ Quotient of Powers and Negative Exponent Properties

$= \frac{1}{4}a^{-5}b^0c^5$ Simplify.

$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$ Negative Exponent and Zero Exponent Properties

$= \frac{c^5}{4a^5}$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{2^2}{2^{-3}}$

2. $\frac{m}{m^{-4}}$

3. $\frac{(-x^{-1})^0}{4w^{-1}y^2}$

4. $\frac{(a^2b^3)^2}{(ab)^{-2}}$

5. $\frac{x^4y^0}{x^{-2}}$

6. $\frac{(6a^{-1}b)^2}{(b^2)^4}$

7. $\left(\frac{4m^2n^2}{8m^{-1}\ell}\right)^0$

8. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$

NAME ______________________________ DATE ____________________ PERIOD __________

Study Guide *Simplifying Radical Expressions*

7

Product Property of Square Roots The **Product Property of Square Roots** and prime factorization can be used to simplify expressions involving irrational square roots. When you simplify radical expressions with variables, use absolute value to ensure nonnegative results.

Product Property of Square Roots	For any numbers *a* and *b*, where $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example 1: Simplify $\sqrt{180}$.

$\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}$	Prime factorization of 180
$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{5}$	Product Property of Square Roots
$= 2 \cdot 3 \cdot \sqrt{5}$	Simplify.
$= 6\sqrt{5}$	Simplify.

Example 2: Simplify $\sqrt{120a^2 \cdot b^5 \cdot c^4}$.

$\sqrt{120a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^3 \cdot 3 \cdot 5 \cdot a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{a^2} \cdot \sqrt{b^4 \cdot b} \cdot \sqrt{c^4}$

$= 2 \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot |a| \cdot b^2 \cdot \sqrt{b} \cdot c^2$

$= 2|a|b^2c^2\sqrt{30b}$

Exercises

Simplify each expression.

1. $\sqrt{28}$

2. $\sqrt{162}$

3. $\sqrt{2} \cdot \sqrt{5}$

4. $\sqrt{4a^2}$

5. $\sqrt{300a^4}$

6. $4\sqrt{10} \cdot 3\sqrt{6}$

7. $\sqrt{24a^4b^2}$

8. $\sqrt{150a^2b^2c}$

Study Guide *Simplifying Radical Expressions (continued)*

8

Simplifying Radical Expressions

Quotient Property of Square Roots A fraction containing radicals is in simplest form if no radicals are left in the denominator. The **Quotient Property of Square Roots** and **rationalizing the denominator** can be used to simplify radical expressions that involve division. When you rationalize the denominator, you multiply the numerator and denominator by a radical expression that gives a rational number in the denominator.

Quotient Property of Square Roots	For any numbers a and b, where $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Example: Simplify $\sqrt{\frac{56}{45}}$.

$\sqrt{\frac{56}{45}} = \sqrt{\frac{4 \cdot 14}{9 \cdot 5}}$ Factor 56 and 45.

$= \frac{2 \cdot \sqrt{14}}{3 \cdot \sqrt{5}}$ Simplify the numerator and denominator.

$= \frac{2\sqrt{14}}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$ to rationalize the denominator.

$= \frac{2\sqrt{70}}{15}$ Product Property of Square Roots

Exercises

Simplify each expression.

1. $\frac{\sqrt{9}}{\sqrt{18}}$

2. $\frac{\sqrt{100}}{\sqrt{121}}$

3. $\frac{8\sqrt{2}}{2\sqrt{8}}$

4. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{5}{2}}$

5. $\sqrt{\frac{3a^2}{10b^6}}$

6. $\sqrt{\frac{100a^4}{144b^8}}$

7. $\frac{\sqrt{4}}{3 - \sqrt{5}}$

8. $\frac{\sqrt{5}}{5 + \sqrt{5}}$